

Graph products for crease patterns and truss frameworks of origami structures

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Abstract

Origami has provided various interesting applications in science and engineering. As shown in Fig. 1, the four-fold rigid origami with Miura pattern is a classic flat-foldable tessellation with periodic and parallel creases, which retains a single degree-of-freedom during folding. In fact, the secret of the intriguing properties of origami is proper crease pattern in the 2D sheet. Thus, design of crease patterns is important for developing an innovative origami with desired characteristics. However, this is a big challenge encountered by not only origami artists and also scientists and engineers who introduce origami into various fields.

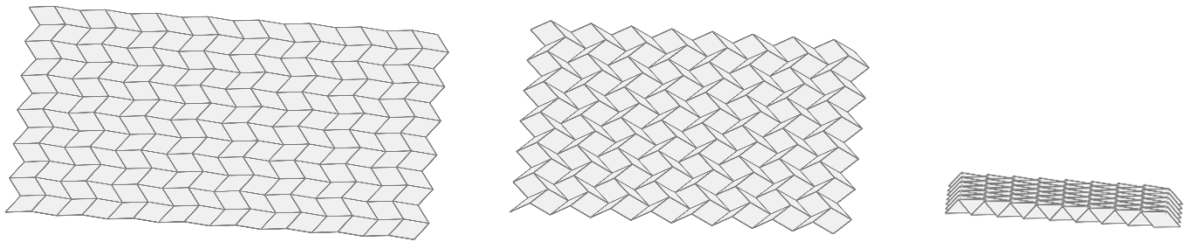


Figure 1: Rigid folding of an origami with Miura pattern.

As most practical origami structures contain repeated unit cells, graph products provide a suitable choice for the formation of crease patterns. Here, we will employ the undirected and directed graph products for the crease patterns and truss frameworks of origami structures.

Notably, a crease pattern of origami can be considered to be a set of directionless creases which satisfy the foldability condition, and this pattern can be exactly expressed by specific graph product of independent graphs. For instance, Fig. 2(a) shows the corresponding crease pattern of the Miura-ori illustrated in Fig. 1, which is the Cartesian product of a straight line P_{17} and a periodic polylines G_1 . The truss framework the Miura-ori shown in Fig. 2(b) is the strong Cartesian product of P_{17} and G_1 .

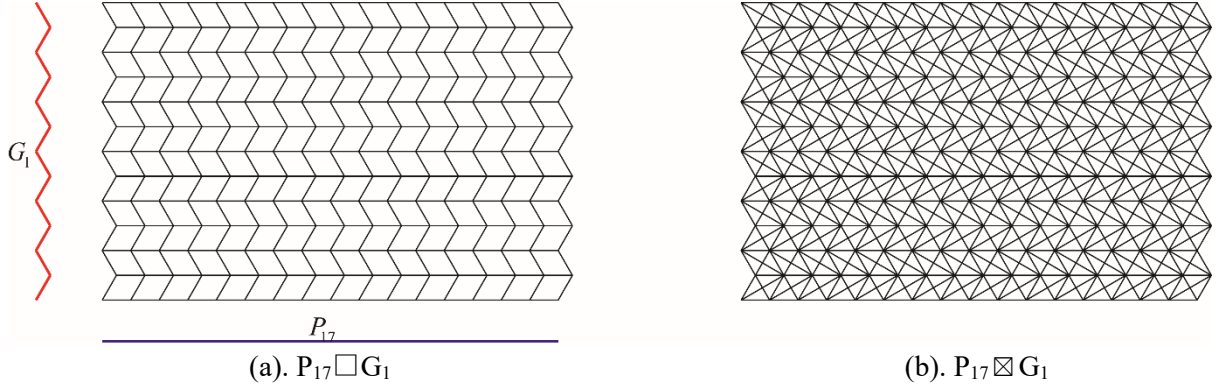


Figure 2: Crease pattern and truss framework of Mirua-ori respectively denoted by: (a) the Cartesian product, and (b) the strong Cartesian product of two graphs.

Moreover, directed graph products are presented in this study (e.g., the patterns shown in Fig. 3), whereas the operators for the union and ring sum of these products are described and utilized.

It turns out that the present method can be effectively implemented in the formation of different patterns of origami, and also convenient for constructing involved matrices and simplified structural models of origami structures. Furthermore, our method can be extended to develop general origami pattern (see Fig. 3 and Fig. 4).

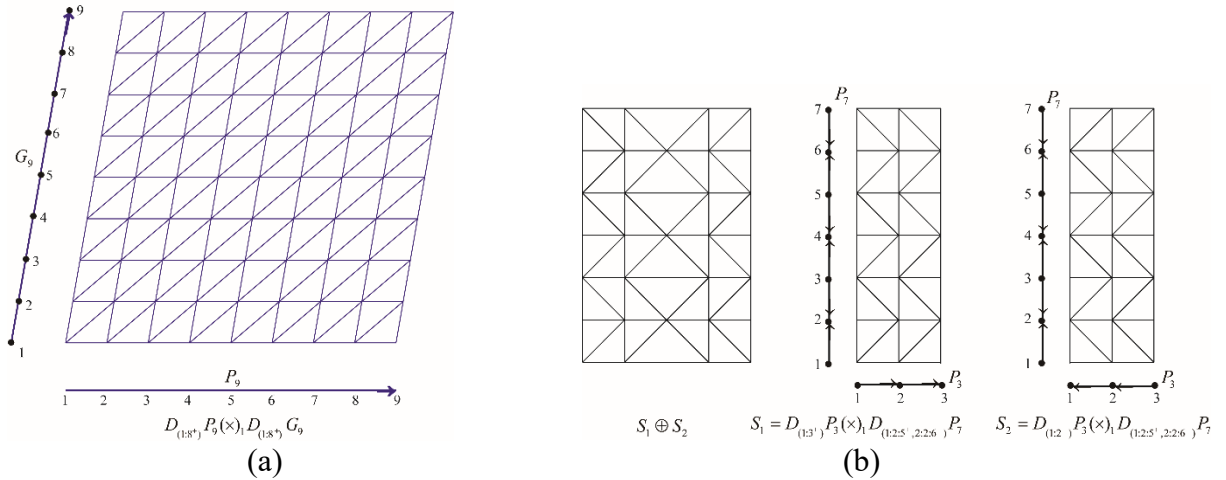


Figure 3: Crease patterns of multi-fold origami expressed by directed graph products.

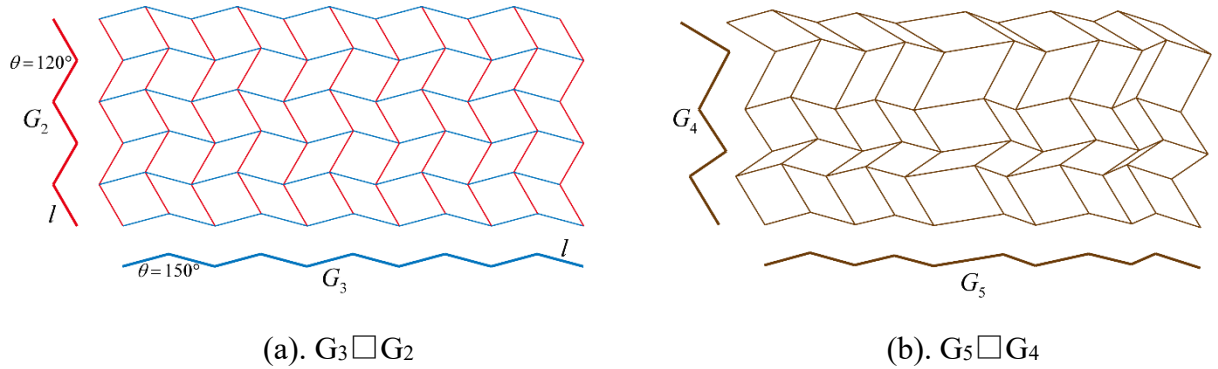


Figure 4: Crease patterns of two general four-fold origami obtained from the Cartesian products of sub-graphs.